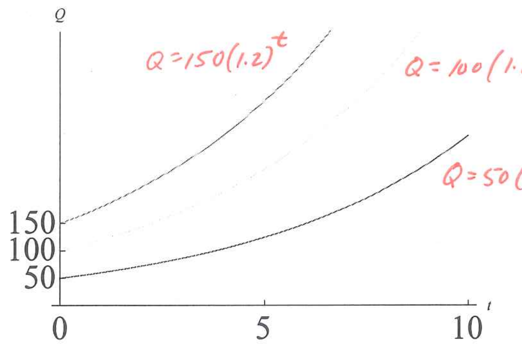
Sec. 4.3 Graphs of Exponential Functions

Graphs of the Exponential Family: The Effect of the Parameter a

In the formula $Q = ab^t$, the value of a tells us where the graph crosses the Q-axis, since a is the value of Q when $t = 0$. Match each equation to the appropriate graph.



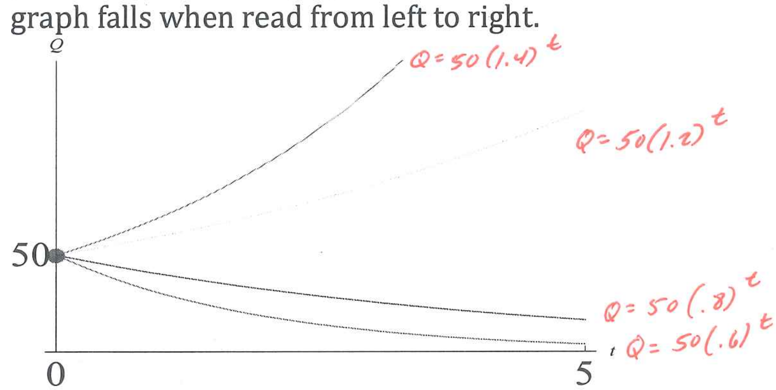
EQUATIONS:

$Q = 150\,(1.2)^t$

$Q = 50\,(1.2)^t$

$Q = 100\,(1.2)^t$

The growth factor, b, is called the *base* of an exponential function. Provided a is positive, if $b > 1$, the graph climbs when read from left to right, and if $0 < b < 1$, the graph falls when read from left to right.



EQUATIONS:

$Q = 50\,(.8)^t$
$Q = 50\,(1.4)^t$
$Q = 50\,(1.2)^t$
$Q = 50\,(.6)^t$

The horizontal line y = k is a horizontal asymptote of a function, f, if the function values get arbitrarily close to k as x gets large (either positively or negatively or both). We describe this behavior using the notation

$$f(x) \to k \text{ as } x \to \infty$$

or

$$f(x) \to k \text{ as } x \to -\infty.$$

Look at graphs

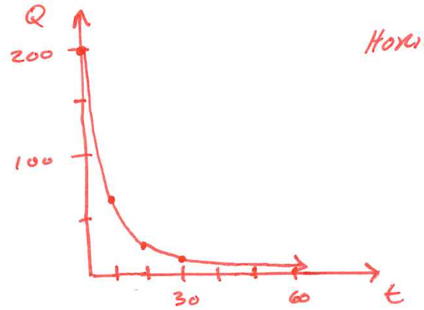
Alternatively, using limit notation, we write

$$\lim_{x\to\infty} f(x) = k \quad \text{or} \quad \lim_{x\to-\infty} f(x) = k$$

Ex. A capacitor is the part of an electrical circuit that stores electric charge. The quantity of charge stored decreases exponentially with time. Stereo amplifiers provide a familiar example: When an amplifier is turned off, the display lights fade slowly because it takes time for the capacitors to discharge. If t is the number of seconds after the circuit is switched off, suppose that the quantity of stored charge (in micro-coulombs) is given by $Q = 200(0.9)^t$, $t \geq 0$.

a. Describe in words how the stores charge changes over time. It decreases by 10% each second
b. What quantity of charge remains after 10 seconds? 20 seconds? 30 seconds? 1 minute? 2 minutes? 3 minutes?
c. Graph the charge over the first minute. What does the horizontal asymptote of the graph tell you about the charge?

$Q = 200(.9)^{10} = 69.736$
$= 200(.9)^{20} = 24.315$
$= 200(.9)^{30} = 8.478$
$= 200(.9)^{60} = .359$
$= 200(.9)^{120} = .000646$
$= 200(.9)^{180} = .00000116$



HORIZONTAL ASYMPTOTE

$Q = 0$

As time increases, the charge approaches zero.

After 60 seconds, it is basically zero.

Ex. A 200 g sample of carbon 14 decays according to the formula $Q = 200(.886)^t$ where t is in thousands of years. Estimate when there is 25 g of carbon 14 left.

Graphically: Intersect with $y = 25$ SOLVE $25 = 200(.886)^t$

$t = 17.180$

$\boxed{t = 17{,}180 \text{ years}}$

Ex. Given the following table of population data for the city of Houston since 1900, find an exponential equation to model the situation. Graph your solution. What is the limit?

t	N	t	N
0	184	60	1583
10	236	70	2183
20	332	80	3122
30	528	90	3733
40	737	100	4672
50	1070	110	5937

Exponential Regression (#10)

$N = 190(1.034)^t$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = 0$

Ex. Beth is interested in finding a function that explains the closing price of Harley Davidson stock at the end of each year. She obtains the following data:

	Year (x)	Closing Price (y)
	1987 (x = 0)	.392
1	1988	.7642
2	1989	1.1835
3	1990	1.1609
4	1991	2.6988
5	1992	4.5381
6	1993	5.3379
7	1994	6.8032
8	1995	7.0328
9	1996	11.5585
10	1997	13.4799
11	1998	23.5424
12	1999	31.9342
13	2000	39.7277

a. Using a graphing calculator, draw a scatter diagram with year as the independent variable.

b. Using a graphing calculator, fit an exponential function to the data. $y = .5532(1.4028)^x$

c. Graph the exponential function found in part b on the scatter diagram.

d. Using the solution to part b, predict the closing price of Harley Davidson stock at the end of 2001. $(14 = x)$

$y = .5532(1.4028)^{14}$

CALC-VALUE

\$ 63.2154

HW: pg 152 – 155, #1, 3, 5, 10, 11-14, 15, 17, 28, 29, 30, 38, 39, 40, 41, 44